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RADIATIVE - CONDUCTIVE HEAT TRANSFER UNDER
CONDITIONS OF A REGULAR MODE OF THE FIRST
KIND

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The regularities of heating a plane layer of a semitransparent condensed medium with optically smooth surfaces are analyzed under the conditions of a regular mode of the first kind.

The question of the existence of regular modes in partially transparent materials was studied in [1, 2]. It was shown in these papers that regularization of the temperature field sets in for a linear change in the temperature of the layer boundaries, just as this is observed in the classical theory of heat conduction. The present paper is devoted to a theoretical investigation of the regularization process for radiative-conductive heat transfer under conditions of a constant coefficient of convective heat transfer and constant temperature of the environment when the radiant heat flux in the plate is equal to or exceeds the heat flux because of molecular heat conduction.

To clarify the dynamics of temperature field development under the combined heat-transport mechanism, let us consider the problem in the following formulation. Let a semitransparent plate with optically smooth surfaces, within which heat transport is subject to the Fourier general law, have a uniform temperature distribution $T(0, x) = T^0$ at the initial time $\tau = 0$. The plate is placed in a plane channel (Fig. 1) over which flows a hot gas whose temperature T_g and convective heat-transfer coefficient h_g are given and constant in time. The channel walls are force cooled and their emissivity is one. The thermophysical and radiation characteristics of a partially transparent layer are independent of the temperature.

The process of combined heat transport by radiation and heat conduction is described by a coupled system of essentially nonlinear integrodifferential equations [3]

$$\cos \theta \frac{\partial \Phi^\pm}{\partial x} = \mp \alpha \Phi^\pm \pm \alpha n^2 B(\lambda, T) / \pi, \quad (1)$$

$$C_p \frac{\partial T}{\partial \tau} = K \frac{\partial^2 T}{\partial x^2} + 2 \int_{(\lambda_0)} d\lambda \int_0^{\pi/2} \alpha [\pi (\Phi^+ + \Phi^-) - 2n^2 B] \sin \theta d\theta. \quad (2)$$

Since the emissivity of the surfaces bounding the channel is one, then in practice all the radiation incident is absorbed and none reflected. The intrinsic radiation of the channel surface and the gas can be neglected since the channel walls are cooled, and the optically thick gas layer is considered small. Therefore, the ex-

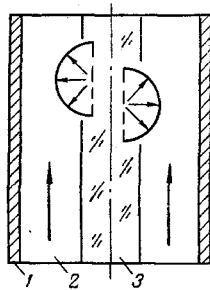


Fig. 1

Fig. 1. Heating diagram of a semitransparent plate in a plane channel: 1) semitransparent plate; 2) hot gas flux; 3) cooled channel wall.

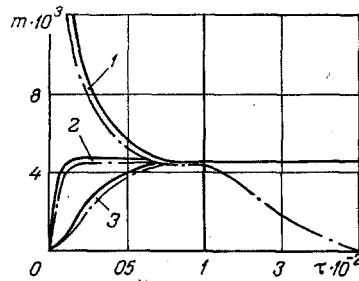


Fig. 2

Fig. 2. Change in the heating rate m (sec^{-1}) in time τ (sec) for different plate sections: 1) $\xi = 0$; 2) 0.2; 3) 0.5.

ternal radiation flux on the semitransparent plate cannot be taken into account.

We write the boundary conditions on the surface $x = 0$ for the limit case under consideration in the form

$$\Phi^+ - R\Phi^- = 0, \quad (3)$$

$$K(\partial T/\partial x) + h_e(T_e - T) - \int_{(\lambda_0)} \epsilon B(\lambda, T) d\lambda = 0. \quad (4)$$

From symmetry conditions, we have for the middle of the plate $x = d/2$

$$\Phi^+ = \Phi^-, \quad \partial T/\partial x = 0. \quad (5)$$

The reflection coefficient from the inner layer surfaces depends on the angle or ray incidence θ and is determined by the Fresnel formulas. The third term in the boundary condition (4) is written under the assumption that the intrinsic surface radiation is subjected to Lambert's law in the spectral domain (λ_0) .

The problem is to see the functions $T(x, \tau)$ and $\Phi^\pm(x, \lambda, \theta)$ satisfying (1) and (2) and the boundary conditions (3)–(5) with the initial condition $T(x, 0) = T^0(x)$ for the temperature field.

Underlying the solution is the finite-difference method. In conformity with this, the general scheme of the numerical solution appears as follows. The time range of interest is partitioned into a number of equal intervals of magnitude $\Delta\tau$. The derivatives are written in finite differences and the problem is reduced to the successive determination of the desired functions at discrete points in time with the spacing $\Delta\tau$.

At the initial time $\tau = 0$ for which the temperature distribution is given, the distribution of the intensities $\Phi^+(x)$ and $\Phi^-(x)$ is found for all the necessary values of the variables λ and θ by numerical integration of the transport equations with the appropriate boundary conditions.

We use iteration to solve the coupled system (1) and (2) for the next time $\tau = \Delta\tau$. We start the iteration by seeking the temperature field over the layer thickness when we use the solution referring to the preceding time as the zeroth approximation for the functions Φ^\pm in the generalized Fourier equation. We then find the first approximation for the functions Φ^\pm from the temperatures found by solving the transport equations. The iterations are repeated until the given condition of convergence of the calculational process in the temperatures and the radiant energy fluxes is satisfied. Both boundary-value problems are solved by using an algorithm of the factorization method which is elucidated in detail in [4].

Let us turn to the results of the numerical analysis. Curves of the change in the heating rate $m = -\partial \ln |T - T_e|/\partial \tau$ characterizing the rate of layer heating in time are presented in Fig. 2 for three plate sections $\xi = x/d$, obtained with different factors taken into account. The solid lines describe the nature of the change in $m(\tau)$ for an ordinary material when the heat transfer is realized just by molecular heat conduction. Another example of the change in $m(\tau)$ is shown by the dash-dot lines. These results correspond to a semitransparent material with an optical thickness $\alpha d = 1$ for the layer when the absorption coefficient α and the refraction coefficient n are constant in the whole spectrum range (λ_0) and the intrinsic surface emission

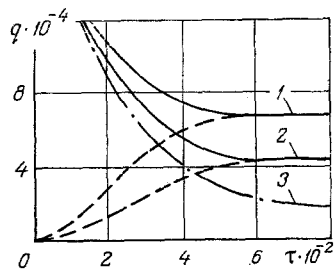


Fig. 3

Fig. 3. Change in the integrated radiant and convective heat fluxes q (W/m^2) during heating τ (sec): 1) $\alpha d = 1$; 2) 0.2; 3) convective heat flux in the case of pure heat conduction.

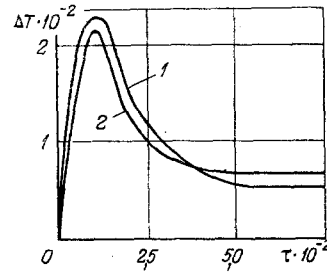


Fig. 4

Fig. 4. Dependence of the temperature drop $\Delta T = T(0) - T(d/2)$ during heating for different optical layer thicknesses: 1) $\alpha d = 2$; 2) 0.2

$\varepsilon = 0$ is neglected in the range of strong absorption of the substance. Computations were executed for the following values of the parameters: $d = 2 \cdot 10^{-2}$ m, $\lambda_0 = (0.25-4.8)$ μm , $K = 1.65$ $\text{W}/\text{m} \cdot \text{deg}$, $C_p = 2.14$ $\text{J}/\text{m}^3 \cdot \text{deg}$, $h_e = 125$ $\text{W}/\text{m}^2 \cdot \text{deg}$, $n = 1.5$, $T^\circ = 293^\circ\text{K}$, $T_e = 1400^\circ\text{K}$.

It is seen from a comparison of the curves presented that three distinct stages can be extracted during the heating depending on the nature of the change in the quantity $m(\tau)$. For small values of τ the heating rate of the different layers turns out to be distinct (irregular stage) and since the general temperature level is sufficiently low, then the dash-dot and solid lines practically agree here. In the second stage of the heating, regularization of the temperature field sets in which is characterized by $m(\tau) = \text{const}$ for the different layers of a plate of transparent and opaque materials, where their heating rate continues to remain identical for time intervals of not too long duration. Then as the general temperature level rises, the nature of the dependence $m(\tau)$ becomes perfectly different for the cases mentioned. If the heating rate is independent of the time in the case of pure heat conduction, then m depends essentially on τ for the combined heat-transfer mechanism. In this stage m tends asymptotically to zero for a semitransparent material. The differences in the heating rate for the different layers turn out to be negligible in the considered range of optical layer thicknesses $\alpha d = 0.2-2$. Thus, for $\alpha d = 1$ the difference between the heating rates for the middle and the surface of the plate is not more than fractions of a percent and can be neglected.

Hence, in the parameter range considered the temperature field regularization for a semitransparent material is observed in the coordinate but not in the time.

Curves of the change in the integrated radiant and convective heat flux during heat transfer to the environment on one of the surfaces are represented in Fig. 3 for different optical layer thicknesses. The dashed lines correspond to the radiant flux lost by the plate, and the solid lines to the heat flux supplied because of convective. The dash-dot line characterizes the convective heat flux in the case of pure heat conduction of the material. It is seen from the figure that the convective heat flux decreases to zero in time in the case of pure heat conduction, while it tends to a certain constant value determined by the integrated emissivity of the plate for the combined heat-transfer mechanism. Stabilization of the radiant and convective heat fluxes in time occurs sufficiently rapidly and is accelerated somewhat as the optical thickness of the layer increases.

The dependence of the temperature drop between the surface and the middle of the plate during heating is shown in Fig. 4 for different optical thicknesses of the layer. As is seen, the maximum temperature drop is observed in the initial stage of the heating and then decreases monotonically, tending asymptotically to some constant different from zero. Therefore, in contrast to simple heating of ordinary materials, the temperature distribution in a semitransparent plate turns out to be nonuniform upon emergence into the stationary mode, with a minimum at the midpoint. The singularities obtained in the temperature distribution over the layer thickness are due to the absence of any external radiation flux penetrating into the plate, which would be equal in magnitude and equivalent in spectral composition to the flux which the semitransparent plate radiates.

On the whole, the results of the numerical analysis performed indicate that the nature of heating under a combined heat-transfer mechanism in semitransparent materials differs substantially from the regularities of the change in temperature field in purely molecular heat conduction under conditions of a regular mode of the

first kind. In physical respects, the singularities established are a result of the intrinsic volume emission of the substance, which results in the appearance of certain effective volume sources in the plate which are variable in thickness and in time, but in mathematical respects are a result of the nonlinearity of the initial equations.

NOTATION

τ , time; T , temperature; T_e , environment temperature; h_e , coefficient of convective heat transfer; x , coordinate; d , layer thickness; α , coefficient of absorption of the substance; n , refractive index; B , surface emission density of blackbody; C_ρ , volume specific heat; K , coefficient of heat conduction; λ , emission wavelength; (λ_t) , wavelength band where the material is partially transparent; (λ_o) , wavelength band where the material is opaque; ϵ , surface emissivity in the band (λ_t) ; $R(\theta)$, coefficient of reflection from the inner surfaces of the layer; Φ^+ , intensity of beams making acute angles θ with the internal normal to the surface $x = 0$; Φ^- , intensity of beams making acute angles with the internal normal to the surface $x = d$; m , heating rate; ξ , dimensionless coordinate; q , heat flux.

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HEAT-TRANSFER RADIATION IN A SELECTIVE GAS FLOW IMPINGING ON A HEATING SURFACE

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The problem of computing the heat-transfer radiation in a selective gas flux impinging on a heating surface is examined on the basis of the Curtis-Godson approximation and a statistical model of the absorption band. Numerical results are presented for carbon dioxide. Their comparison with the results of a computation on the basis of a grey model showed a substantial difference in the magnitudes of the resultant fluxes on the boundary surfaces.

Heat-transfer radiation in a gas flow impinging on a heating surface has been investigated in [1-4]. The medium [1-3] was hence assumed grey, while the heat transfer in carbon dioxide, whose absorption spectrum was borrowed from [5] in the words of the authors, was examined in [4].

In this paper the heat transfer in a selective gas flow is examined on the basis of a statistical model of the absorption band and the Curtis-Godson approximation.

A gas layer of thickness l is bounded by black surfaces 1 and 2 with the given temperatures T_{c1} and T_{c2} . The gas flow, with the temperature T_0 , enters at surface 1 and moves toward surface 2 (Fig. 1).

This is a one-dimensional problem, the process is stationary, the pressure is constant, and we neglect the temperature dependence of the specific heat of the gas and the heat conduction.

The energy equation and the boundary condition are written in dimensionless form as follows:

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